Pursuing perfection in intraocular lens calculations

IV. Rethinking astigmatism analysis for intraocular lens-based surgery: Suggested terminology, analysis, and standards for outcome reports

We saved our perhaps most controversial topic for this fourth editorial: analyzing astigmatic change following IOL-based surgery, that is, cataract surgery and refractive lens exchange. This topic is challenging for three major reasons:

1. **Range of analytical approaches.** In the January 2001 issue of JCRS, several authors described their methods for analyzing change in corneal astigmatism following laser in situ keratomileusis surgery. Several of these methods are commonly and effectively used in peer-reviewed scientific studies to characterize astigmatic changes.
2. **Analyzing astigmatic change in intraocular lens (IOL)-based surgery is more complex than in corneal refractive surgery.** One issue is simply the transition of terminology and standards for reporting outcomes from corneal laser refractive surgery to toric-IOL-based surgery, which was recently addressed by a joint editorial in the *Journal of Cataract & Refractive Surgery* and the *Journal of Refractive Surgery*.[1] A larger problem is complexity of measurement and analysis. With corneal refractive surgery, we evaluate changes in two measurable entities: refraction and anterior corneal curvature. With IOL-based surgery, however, parameters required to analyze astigmatic change after cataract surgery include: (a) refraction; (b) anterior corneal astigmatism; (c) posterior corneal astigmatism,[2,3] which still eludes consistently accurate measurement; (d) IOL alignment, which must be rigorously measured; (e) effective IOL toricity at corneal plane, which can be calculated but not directly measured; and (f) IOL tilt,[4,5] which induces astigmatism with nontoric and toric IOLs.
3. **There are four clinical scenarios in which cataract surgery alters ocular astigmatism.** They are: (a) nontoric IOL without corneal relaxing incisions, (b) nontoric IOL with corneal relaxing incisions, (c) toric IOL without corneal relaxing incisions, and (d) toric IOL with corneal relaxing incisions. Since both the cornea and IOL have an astigmatic component (tilt in the case of a spherical IOL), for each of these scenarios we are interested in the corneal, IOL-induced, and total ocular astigmatism (which is typically characterized by the manifest refractive astigmatism, but objective measures of ocular astigmatism can also be used).

In this editorial, we propose (1) simplified terminology that can be used for both IOL and corneal-based refractive surgery and (2) a hybrid approach to astigmatism analysis that we feel is clear, comprehensive, and valuable to clinicians and researchers alike while recognizing the merits of other approaches.

SUGGESTED TERMINOLOGY

Traditionally, terminology for data analysis in IOL-based surgery includes terms such as predicted (or attempted/targeted) versus actual (or achieved) refraction. The difference between the actual and the predicted refraction is defined as the prediction error.

We think that the terms "predicted," "actual," and "prediction error" are inherently clear and lend themselves to use in all types of refractive and cataract surgery. With that in mind, we propose the following terminology:

- **Predicted SIA (surgically-induced astigmatism)***: the astigmatic change that the surgery was designed to produce.
- **Actual SIA:** the astigmatic change that the surgery produced.
- **SIA prediction error:** the vector difference between the above 2 terms: (actual SIA) – (predicted SIA).

As was mentioned above and discussed elsewhere,[1,6] in toric-IOL-based surgery, the term *SIA* should be further refined to clarify the source of the induced change in astigmatism. Hence, we would like to suggest the following refined terminology:

- **SIA$_{Cornea}$:** the change in total corneal astigmatism. This change can be induced either by the cataract incisions alone or by additional corneal incisions, such as limbal relaxing incisions and astigmatic keratotomy.

- **SIA$_{IOL}$:** the astigmatic change induced by a toric IOL, due to its toricity, and by either a toric or nontoric IOL, due to tilt and/or decentration.
- **SIA$_{Total}$:** the total astigmatic change induced by the surgery.

We further suggest the use of the phrase "postoperative refractive astigmatism" for analysis of routine IOL surgery with the following terminology:

- **Predicted postoperative refractive astigmatism:** the predicted postoperative refractive astigmatism at the corneal plane.
- **Actual postoperative refractive astigmatism:** the postoperative manifest refractive astigmatism at the corneal plane.
- **Postoperative refractive astigmatism prediction error:** the vector difference between the actual and the predicted postoperative refractive astigmatism.

Change in Corneal Astigmatism

The SIA$_{Cornea}$ is the change in corneal astigmatism induced by the cataract surgical incisions and by any relaxing incisions that are also performed.[1] Estimation of SIA$_{Cornea}$ is essential in surgical planning, whereas postoperative measurement of corneal astigmatism should be used to analyze outcomes of toric IOL surgery.

The predicted SIA$_{Cornea}$ is a vector. Traditionally, SIA for a defined cataract incision is calculated as the mean vector magnitude for the case series, independent of vector angle. However, the meridional direction of these vectors can be highly variable. For example, if SIA$_{Cornea}$ for one eye is 0.4 diopter (D) @ 180 and for another is 0.4 D @ 90, the mean magnitude is 0.4 D, but the mean vector is zero. The mean vector, or *centroid*, accurately incorporates both magnitude and meridian and is the proper value to use. This can be calculated using online tools.[A]

However, there are complexities even for this ostensibly simple calculation:

1. **Case-to-case variability.** Depends on incision features (location, architecture including width and length, stretching) and inherent corneal biomechanical features (corneal radius, thickness, rigidity).
2. **Accuracy of the corneal astigmatism measurements.** As mentioned above, currently there is no accurate method to measure the total corneal astigmatism. Hence, there is no accurate method to measure the actual SIA$_{Cornea}$.
3. **Relatively low repeatability of the corneal astigmatism measurements by standard measuring devices.** Figure 1 shows double-angle plots of the differences in corneal astigmatism measurements (same eyes), in healthy volunteers 5 to 10 days apart, using several measuring devices.[B] In theory, the differences between each pair of astigmatism measurements should be near zero; however, the relatively large 95% confidence

ellipses reflect the limitation in assessing the actual SIA$_{Cornea}$ for an individual case.

4. **Stability of the actual SIA$_{Cornea}$ over time.** Postoperative corneal measurements are typically performed 3 to 4 weeks postoperatively, although corneal stability conceivably may not occur until 3 or 4 months postoperatively.
5. **Recent studies** suggest that the magnitude of the SIA$_{Cornea}$ may vary depending on preoperative astigmatism magnitude and the incision location relative to the corneal steep meridian.

An obvious way to mitigate the variability introduced by SIA$_{Cornea}$ is to measure corneal curvature postoperatively. We believe that this is valuable for studies that evaluate toric IOL outcomes.

This analysis is almost always based solely on measured changes in anterior corneal astigmatism, when in fact we want to know changes in total corneal astigmatism. Until we have validated methods for measuring posterior and hence total corneal astigmatism, changes in total corneal astigmatism will most often be considered equivalent to anterior corneal changes.

Change in Astigmatism Induced by the Toric Intraocular Lens

The SIA$_{IOL}$ refers to the change in astigmatism that is induced by the toric IOL at the corneal plane. The SIA$_{IOL}$ prediction errors are caused by some combination of incorrect estimation of the effective cylinder power of the toric IOL at the corneal plane, toric IOL misalignment, difference in actual versus labeled IOL spherical and toric power, and IOL tilt or decentration.

Toric IOL calculation formulas provide a value for the estimated refractive astigmatism after inserting and precisely aligning a given toric IOL. Until recently, most commercially available toric calculators used a fixed ratio of a given IOL's toricity to refractive astigmatism to calculate the astigmatic effect of a toric IOL at the spectacle plane (fixed ratio toric calculator). For example, if the manufacturer's stated value for IOL toricity is 2.00 D, this value is translated to the spectacle plane. However, a more accurate way is to modify the effective toricity based on the spherical power of the IOL and the predicted effective lens position (ELP)[7] (vergence toric calculator). Table 1 gives examples of the impact of ELP and IOL power on effective IOL toricity.[C] Software that provides this calculation includes the Holladay IOL Consultant,[D] the Barrett toric calculator,[E] and many of the new updated online commercial toric calculators.

To predict the IOL toricity at the corneal plane, a conversion from the spectacle plane to corneal plane is required. To do this, one should first convert the predicted postoperative refraction at spectacle plane to a cross-cylinder format and then vertex each of the two values using this formula: Power$_{Corneal Plane}$ = (Power$_{Spectacle Plane}$) / (1 − [Power$_{Spectacle Plane}$] × [vertex distance in m]). Then, the difference between these two is the correct value for effective toricity at the corneal plane: SIA$_{IOL\ Corneal\ Plane}$.
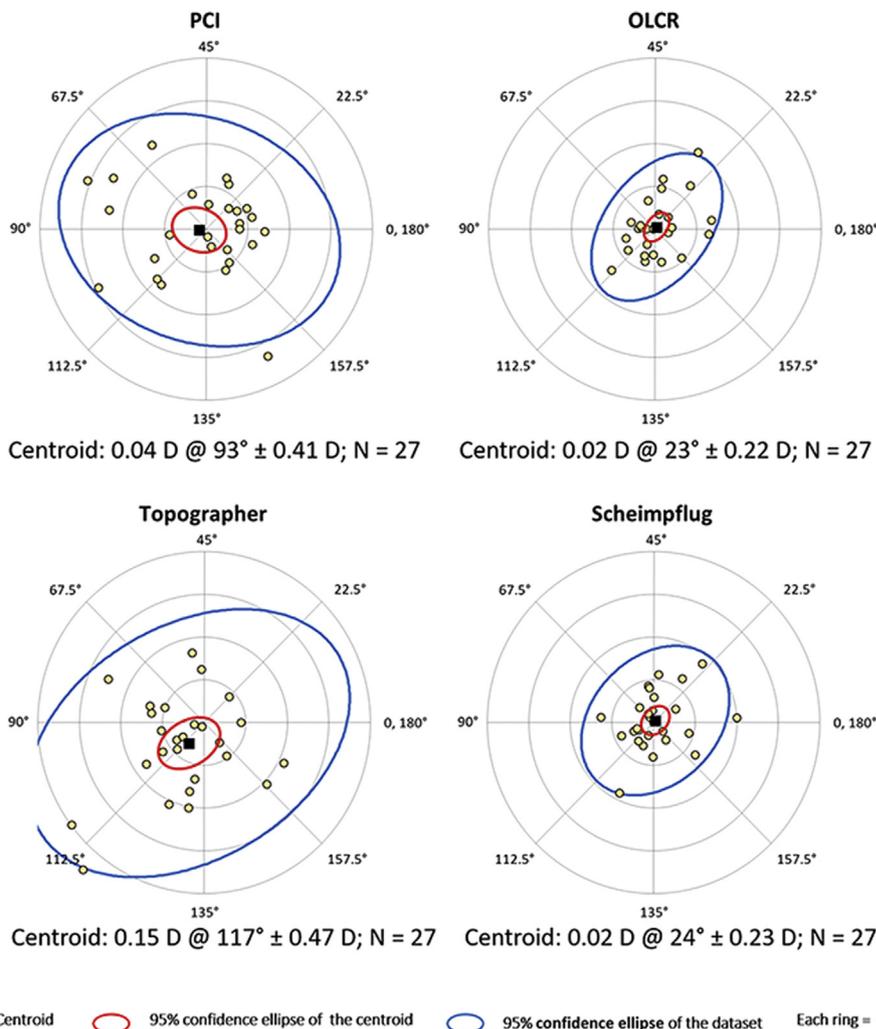
Figure 1. Double-angle plots of the differences in corneal astigmatism measurements (same eyes), in healthy volunteers (n = 27), 5 to 10 days apart, using several measuring devices (OLCR = optical low-coherence reflectometry; PCI = partial coherence interferometry).

Total Induced Changes in Refractive Astigmatism

Ultimately, we want to know how the actual change in refractive astigmatism compares to the predicted change. The predicted change in refractive astigmatism is the vector sum of the corneal and IOL-induced changes: Predicted SIA_{total} = Predicted SIA_{Cornea} + Predicted SIA_{IOL}.

Determining an accurate value for actual SIA_{Total} remains an imperfect science. Typically, it is calculated as the difference between the actual postoperative refractive astigmatism and the preoperative corneal astigmatism. Ideally, it would be calculated as the sum of the actual SIA_{Cornea} and the actual SIA_{IOL}, values that can only be estimated as noted above.

Single-Angle or Double-Angle Plots?

Because astigmatism is a vector, possessing both a magnitude and an orientation, correct astigmatism analysis requires doubling the angle to transform the astigmatism data into 360-degree Cartesian coordinates. One can display this as single-angle (as proposed in the Alpins method[12]) or double-angle plots. We recommend the latter. Since astigmatism vector calculations are performed after doubling the angle, these data can be inserted directly into standard scatter plots that are in fact double-angle plots. Interpreting double-angle plots is initially not as intuitive as single-angle plots, as the latter are familiar to us in our offices (ie,

Table 1. Impact of ELP and IOL power on the required IOL toricity to correct 2.00 D of corneal astigmatism.*

ELP (mm)	4.0	4.5	5.0	5.5	6.0	6.5
IOL power 10 (D)	2.718	2.848	2.988	3.141	3.308	**3.490**
IOL power 22 (D)	2.554	2.659	2.774	**2.900**	3.038	3.190
IOL power 34 (D)	**2.396**	2.477	2.568	2.668	2.780	2.904
IOL power 46 (D)	2.242	2.302	2.369	2.446	2.533	2.631

Bold values are typical values for short, medium, and long eyes.
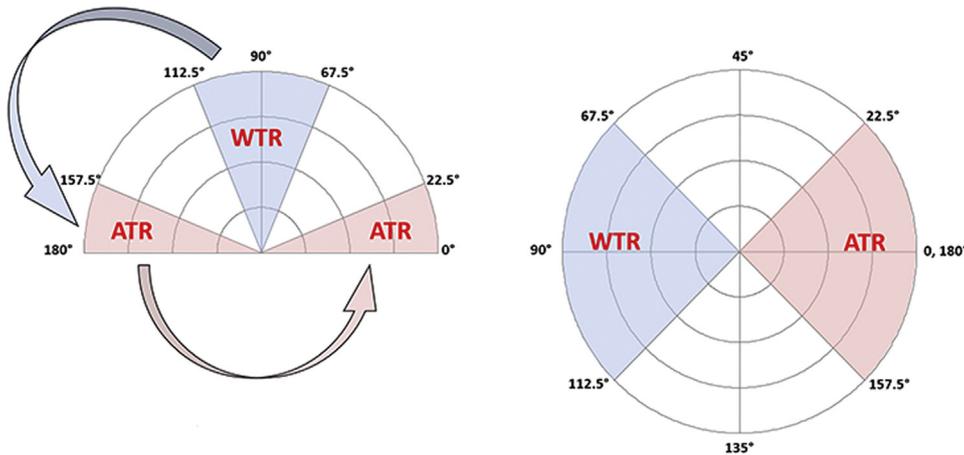ELP = effective lens position; IOL = intraocular lens
*Data from Table 2 in reference [C]

Figure 2. Single-angle plot versus double-angle plot. In a double-angle plot, the WTR eyes are grouped together on the left side of the figure and the ATR eyes are grouped together on the right (ATR = against-the-rule; WTR = with-the-rule).

phoropters). However, the concept of double-angle plots can be easily understood as illustrated in Figure 2.

We think that there are three key advantages of double-angle plots:

1. **The *x* and *y* scatterplot of the data on a double-angle plot maintains the spatial relationship of each astigmatic value.** Take, for example, 1.0 D at 001 degree and 1.0 D at 179 degrees. They should be displayed nearly superimposed, separated by 2 degrees. This is done appropriately with double-angle plots, whereas with single-angle plots they appear on opposite sides of the origin, separated by essentially 2.0 D.
2. **The double-angle plot allows the display of the magnitude and axis or meridian of the average astigmatism (centroid) and the confidence ellipse,** which is the 2-variable analog of the confidence interval for single-variable analyses.
3. **Because double-angle plots group data appropriately,** qualitative assessment of group data is facilitated. One can easily visualize trends in group data and compare them to other datasets.
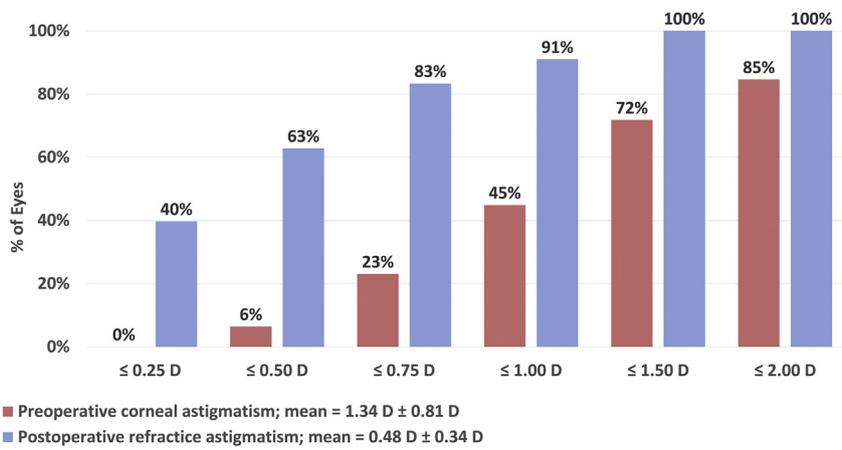
Reporting Outcomes

Ultimately, what do clinicians and researchers need to know about the outcomes of toric IOL surgery? Of course, we need tables showing key demographic data. We will also want to know spherical IOL power accuracy, that is, spherical equivalent outcomes. Data should be analyzed separately for right and left eyes and, for those instances when right and left eye data are combined, astigmatic data from the left eye should first be mirrored about 90 degrees to retain the correct nasal/temporal orientation. To analyze astigmatic outcomes, we propose that, at a minimum, the following questions must be answered and we suggest the following graphs to provide the answers:
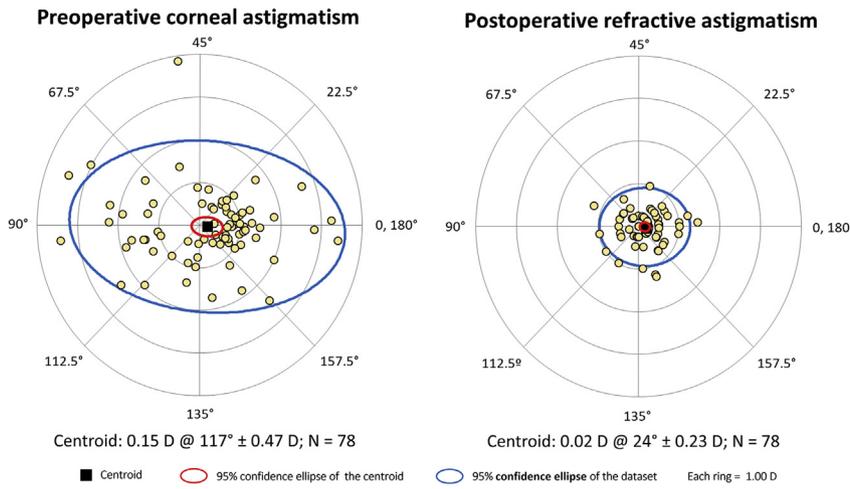
1. **How much astigmatism was present before and after toric IOL implantation?** Obviously, this is fundamental since the magnitude of the postoperative refractive astigmatism determines the patient's visual outcome. To display the data, we recommend Figure 3, *A*, a

cumulative histogram of the magnitude of the preoperative corneal and postoperative refractive astigmatism, with the latter vertexed to the corneal plane. Depending on the data sets, possible bins would be 0.25 D, 0.50 D, 0.75 D, 1.0 D, 1.50 D, and 2.0 D. In addition, one could insert a table showing the means and standard deviations (SD) of the magnitudes of these astigmatism values.
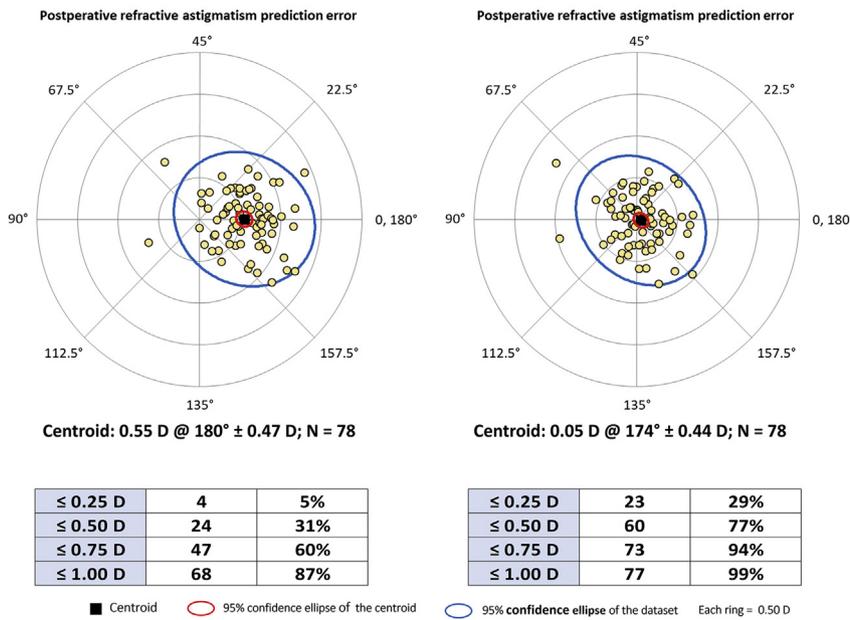
2. **What are the preoperative and postoperative astigmatic vectors and their means and spread?** To display this, we recommend Figure 3, *B*, double-angle plots with means (ie, centroid), SDs of the centroids, and 95% confidence ellipses of preoperative corneal and postoperative refractive astigmatism and of the centroids:

   - The centroid is the vectoral center of the data.
   - The total SD of the centroid is one indicator of the spread of the data around it. To calculate the SD of centroid for the bivariable *x* and *y*, one must first calculate the total variance, which is the sum of partial variances of *x* and *y*. The total SD of centroid is the square root of the total variance.
   - The 95% confidence ellipse is the boundary that includes 95% of the observations (N) in the dataset. The *x* and *y* semidiameters of this ellipse are rotated to the correct orientation and have their center at the centroid of the data. This confidence ellipse boundary is *analogous* to the 95% confidence interval of the observations in a univariate analysis, which is 1.96 times the SD for a normal distribution.
   - The 95% confidence ellipse semidiameters of the rotated ellipse may be divided by the square root of the number of observations. The smaller ellipse is called the 95% confidence ellipse of the centroid and represents the 95% confidence ellipse boundary of the centroid. This is *analogous* to the 95% confidence interval of the mean in a univariate analysis and is often referred to as 1.96 times the standard error of the mean for a normal distribution.

3. **How accurate were the toric IOL calculation formulas?** To display these data, we recommend Figure 3, *C*, double-angle plots of prediction errors of the formulas being evaluated with centroids, SD of the centroids,

Figure 3. *A*: Cumulative histogram of the magnitude of the preoperative corneal and postoperative refractive astigmatism, vertexed to the corneal plane (n = 78). Means and SDs are also shown. *B*: Double-angle plots of the preoperative corneal and the postoperative refractive astigmatism (n = 78). Centroids and SDs are also shown. *C*: Double-angle plots of postoperative refractive astigmatism prediction errors (n = 78) of a standard toric calculator with no adjustments and adjusted by the Abulafia-Koch formula.[8] Also shown are the percentages of eyes with prediction errors less than or equal to 0.25 D, 0.50 D, 0.75 D, and 1.0 D (SD = standard deviation).

and 95% confidence ellipses of the prediction error(s). One could include a table that shows the percentage of eyes with prediction errors less than or equal to 0.25 D, 0.50 D, 0.75 D, and 1.0 D.

We have prepared an Excel (Microsoft Corp.) spreadsheet that will be made available on the ASCRS website. This spreadsheet will allow investigators to generate Figure 3, *A* as displayed in this editorial and

Figures 3, B and C with centroids and the 95% confidence ellipse.

For statistical analysis, as described by Næser,[9] bivariate analysis of the data is required. The Hotelling's T-squared test should be used, which is the bivariate analog of the 2 sample *t* test used in univariate statistics.[9] This can be accessed in a number of statistical software programs.

Additional analyses can be added at the investigators' discretion to highlight any findings that are unique to their datasets. Separate analysis can be performed on means and SDs of the *x* and *y* Cartesian coordinates[10] or other vector components such as the power vectors J0 and J45.[11] However, these data must be rotated to the appropriate new axes using the multiple R as described by Næser[9] to accurately describe the bivariate analysis of the data.

In this guest editorial, we highlighted some of our thoughts and suggestions regarding astigmatism analysis for IOL-based surgery. Our goal was to provide a method of analysis that is clear, succinct, and optimal for displaying these data.

We would like to acknowledge the tremendous contributions of many pioneers in this field, particularly Næser,[9] whose thesis is a landmark article in this area, but also (in alphabetical order) Alpins,[12–14] Harris,[15] Holladay et al.,[10] Kaye and Patterson,[16] Naeser and Hjortdal,[17] Thibos and Horner,[11] and many others.

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